Choosing Among Diverse Efficiency Measures

Jay E. Ryu

Abstract:
Nonparametric two-stage Conditional Frontier Approach (CFA) obtains true efficiency scores for decision-making units (DMUs) by distinguishing between heterogeneous environmental factors of monitoring force and hostile/favorable production settings. In typical local governments, however, some factors are a *mix* of the two factors and estimated efficiency scores of DMUs might be biased. While the CFA is the latest development in obtaining potentially true efficiency scores of DMUs, it is still unclear about how to identify and address mixed environmental factors. CFA results based on the data for local Ohio school districts show that efficiency scores obtained from the nonparametric two-stage CFA, which controls for the mixed environmental factors only, are still biased. They indicate that we should not use the *mixed* environmental factors *alone* in the first stage of the CFA analysis. Instead, we should adjust for hostile/favorable production settings at the minimum when we apply CFA as the separability condition itself implies.

Key Words: Conditional Frontier Analysis, production efficiency, input output analysis, separability test
JEL Codes: D24, I18

1 Introduction

The studies by Debreu (1951) and Farrell (1957) introduced frontier approaches to measure the productive efficiency of decision-making units (DMUs). Over the years, various methods like nonparametric Data Envelopment Analysis (DEA) and parametric Stochastic Frontier Approaches (SFA) have been used to estimate the managerial efficiency of both private and public organizations. However, these methods often yield divergent efficiency measures for the same DMUs, leading to confusion among researchers and policymakers when comparing productivity. Many studies have tried to address this issue by accounting for non-discretionary environmental factors that impact DMU's operations, aiming to estimate true efficiency scores by whitening out the influence of these factors. Despite these efforts, efficiency scores still tend to diverge even after adjusting for environmental factors, creating further confusion (Pevcin, 2014; Theodoridis & Anwar, 2011; Welde & Odeck, 2011; Odeck, 2007; Farsi & Filippini, 2005; Mortmier, 2002; Bifulco & Bretschneider, 2001; Bifulco & Bretschneider, 2003; Ruggiero, 2003; Chakraborty & Blackburn, 2013; Chakraborty & Poggio, 2008; Ruggiero & Vitaliano, 1999; Huguenin, 2015; Banker & Morey, 1986; Ray, 1991).

One reason for the lack of convergence in efficiency scores is the failure to model whether non-discretionary environmental factors directly affect DMUs' efficiency or if they only shift production sets or frontiers for these units. This distinction is essential as it has significant implications for adjusting environmental factors. To address this, a recently developed test provides a breakthrough by clarifying the difference between these two types of environmental factors. This test, known as a separability test, distinguishes between the effects of these factors on production processes. If the factors affect the distribution of efficiencies, the usual two-stage OLS regression model can adjust for them. However, if they impact the production sets or frontiers, first-stage efficiency scores become meaningless, and Conditional Frontier Analysis (CFA) based on adequate nonparametric two-stage regression adjustment is necessary (Daraio &

Simar, 2007a; Daraio & Simar , 2007b; Daraio, Simar, & Wilson, 2018; Bădin, Daraio, & Simar, 2019; Simar & Wilson, 2011; Bădin, Daraio, & Simar, 2012; Simar, Vanhems, & Van Keilegom, 2016).

Despite the clarification made possible by the separability test and the CFA, however, they fail to realize that some factors under typical managerial settings are likely to be a mix of the two types and as a result, researchers and especially policy makers might easily mistake the mixed environmental factors for purely hostile/favorable environmental factors. In local governments such as K-12 school districts, median voter's tax burden and wealth directly affect the level of school service delivery via local voting mechanism (Eom *et al*., 2014). Tax price and wealth function as monitoring force over school district administration and teaching or school district efficiency, which might be reflected in voting results. At the same time, however, the two factors also reflect the fiscal capacity of school districts: high tax burden as well as taxpayers' wealth imply larger revenues for school districts. If certain school districts are flush with slack resources, for instance, they are likely operating under favorable school district settings. This case exemplifies favorable environmental factors. As a result, some environmental factors, which can be easily used as environmental factors in frontier analysis, tend to be a *mix* of monitoring force and hostile/favorable environmental factors, especially in local governments.

Empirical findings in this paper indicate that we should not use such *mixed* environmental factors *alone* in the first stage of the CFA analysis. Instead, we should adjust for hostile/favorable production settings at the minimum when we apply CFA as the separability test itself implies.

In summary, this paper provides valuable insights for researchers and policy makers on correctly measuring DMUs' efficiency, especially in the context of school districts. The paper includes sections on two different types of environmental factors, the separability test and the CFA, construction of efficiency measures, and discussions on correlations among these measures, concluding with implications for decision-making.

2 Separability Test and Conditional Frontier Approach (CFA)

Various studies have identified environmental factors that affect the managerial efficiency of DMUs including educational institutions (Lee *et al*., 2019; Li *et al*., 2017; De Witte and López-Torres, 2017; Agasisti and Bonomi, 2014). However, no solid theoretical frameworks have presented a reliable test on how to distinguish between monitoring force and hostile/favorable environmental factors until a separability test fills this gap in the literature.

For random variables (*X*, *Y*, *Z*), $X \in \mathbb{R}_+^p$ is a vector of inputs used to produce a vector of outputs $Y \in \mathbb{R}_+^q$ and $Z \in \mathbb{R}^r$ is a vector of non-discretionary environmental factors that are not under the control of managers (Daraio *et al*., 2018, 2021; Daraio and Simar, 2005, 2007a, 2007b; Mastromarco *et al*., 2022). Let $f_{XYZ}(x, y, z)$ be the joint density of (*X*, *Y*, *Z*), which has the support of $\mathcal{P} \subset \mathbb{R}_+^p \times \mathbb{R}_+^q \times \mathbb{R}^r$. The joint density can be decomposed as (Daraio, Simar, and Wilson, 2018; Bădin, Daraio, and Simar, 2019):

$$f_{XYZ}(x, y, z) = f_{XY|Z}(x, y|z) f_Z(z) \quad \textit{Equation 1}$$

Let Ψ^z be the conditional support of $f_{XY|Z}(x, y|z)$ such that:

$$\Psi^z = \{(X, Y) \mid X \; can \; produce \; Y \; when \; Z = z\} \quad \textit{Equation 2}$$

Let $\mathbb{Z}$ be the support of $\mathrm{f_Z}(z)$ and the production set, which is a set of technically feasible combinations of (*x*, *y*), is defined as (Daraio and Simar, 2007b, p. 15; Daraio *et al*., 2018; Bădin *et al*., 2019):

$$\Psi = \{(x, y) \in \mathbb{R}_+^{p+q} \mid x\ can\ produce\ y\} = \cup_{z \in \mathbb{Z}} \Psi^z \quad Equation\ 3$$

Under the above conditions, the non-discretionary environmental factors, *Z*, can affect the production process of DMUs in one of the following three ways: a) only through Ψ^z, b) only through the conditional density, $\mathrm{f_{XY|Z}(x, y|}z)$, thereby affecting the probability of DMUs to reach their optimal boundary, and c) through both Ψ^z and $\mathrm{f_{XY|Z}(x, y|}z)$. The separability condition requires that *Z* affects the production process of DMUs only through the conditional density, $\mathrm{f_{XY|Z}(x, y|}z)$, with no impacts on its support, Ψ^z, such that (Daraio *et al*., 2018; Bădin *et al*., 2019):

$$\Psi^z = \Psi\ for\ all\ z \in \mathbb{Z} \quad Assumption\ 1$$

Assumption 1 indicates that the joint support of (*X*, *Y*, *Z*) can be written as $\mathcal{P} = \Psi \times \mathbb{Z}$. As a result, the non-discretionary environmental factors, *Z*, do not affect the boundaries of Ψ. Instead, *Z* only affects the distribution of efficiencies. If Assumption 1 (the separability condition) holds, we can obtain typical efficiency measures. For instance, a Farrell-type input-oriented efficiency score can be defined as (Daraio and Simar, 2007b, p. 15):

$$\theta(x, y) = inf\{\theta \mid (\theta x, y) \in \Psi\} \quad Equation\ 4$$

A DEA estimator of the production set Ψ, $\widehat{\Psi_{DEA}}$, can be defined as (Daraio and Simar, 2007b, p. 15):

$$\widehat{\Psi_{DEA}} = \{(x, y) \in \mathbb{R}_+^{p+q} | y \le \textstyle\sum_{i=1}^{n} \gamma_i y_i\, ; x \ge \sum_{i=1}^{n} \gamma_i x_i\, , s.t. \sum_{i=1}^{n} \gamma_i = 1; \gamma_i \ge 0, i = 1, \dots, n\} \quad Equation\ 5$$

We can obtain a DEA estimator of efficiency score, $\widehat{\theta_{DEA}}(x, y)$, by replacing Ψ in $\theta x,y⍰ = inf\{\theta \mid (\theta x, y) \in \Psi\}$ Equation 4 with $\widehat{\Psi_{DEA}}$. We can analyze the behavior of $\widehat{\theta_{DEA}}(x, y)$ as a function of Z and apply the usual two-stage approaches to investigate the impact of Z (Bădin, Daraio, and Simar, 2019; Daraio, Simar, and Wilson, 2018).

In contrast, the non-separability assumption is defined as (Daraio *et al*., 2018):

$$\Psi^z \neq \Psi\ for\ some\ z \in \mathbb{Z} \quad Assumption\ 2$$

If the separability condition does not hold, the boundary of Ψ is unattainable for some DMUs facing unfavorable conditions. Therefore, it is meaningless to measure the distance of a unit (x, y) from the boundary of Ψ since it ignores the heterogeneity introduced by Z. In that case, we should apply the Conditional Frontier Approach (CFA) to obtain efficiency scores of DMUs by conditioning on Z. For instance, the attainable conditional DEA production set when $Z = z$, $\widehat{\Psi_{DEA}^z}$, can be derived as the following, for z_i being in h-neighborhood of z for any symmetric kernel with compact support, $K((z - z_i)/h)$, where *h* is a bandwidth of appropriate size (Daraio and Simar, 2007b, pp. 19-20):

$$\widehat{\Psi_{DEA}^z} = \{(x, y) \in \mathbb{R}_+^{p+q} | y \le \textstyle\sum_{\{i|z-h \le z_i \le z+h\}} \gamma_i y_i; x \ge \sum_{\{i|z-h \le z_i \le z+h\}} \gamma_i x_i\ for\ \gamma_i \ge 0\ such\ that\ \sum_{\{i|z-h \le z_i \le z+h\}} \gamma_i = 1\} \quad Equation\ 6$$

We can obtain a conditional DEA estimator of efficiency score, $\widehat{\theta_{DEA}}(x, y|z)$, by replacing Ψ in Equation 4 with $\widehat{\Psi_{DEA}^z}$.

Simar, Vanhems, & Van Keilegom (2016) further show how to account for unobserved heterogeneity in conditional DEA efficiency scores. Once we can find an observed instrumental variable that is related to either input or output but is independent of the unobserved heterogeneity, we can estimate the heterogeneity. Then, conditional frontier estimates can even control for the hidden heterogeneity as environmental factors. For instance, Daraio *et al*. (2021) develop a measure of *unobserved* (latent) factor of heterogeneity, which might affect the boundary of attainable production set in their study on the activity of European universities. The measure captures what remains from academic staff after accounting for the number of enrolled students. Conditional frontier estimates will become less biased when controlling for some unobserved heterogeneity.

However, the above approach needs some knowledge about the link between instrumental varaibles and inputs/outputs. This process causes extra burden when there are multiple inputs or outputs as in this paper. Bădin, Daraio, & Simar (2012) provide a much simpler and practically more applicable approach that is like the above approach in principle. We can analyze the regression $\mathrm{E}(\widehat{\theta_{DEA}}(X,Y|Z)|Z=z)$ as a function of z, by using a flexible regression model that defines $\mu(z)=\mathrm{E}(\widehat{\theta_{DEA}}(X,Y|Z)|Z=z)$ and $\sigma^2(z)=V(\widehat{\theta_{DEA}}(X,Y|Z)|Z=z)$. We may rewrite:

$$\widehat{\theta_{DEA}}(X,Y|Z)|Z=z) = \mu(z)+\sigma(z)\varepsilon \quad \textit{Equation 7}$$

We can run the second-stage regression of the conditional efficiency scores based on the local constant method to obtain $\mu(z)$. Then, we regress the residuals of the second-stage regression on z to obtain $\sigma(z)$, again based on the local constant method. Finally, we can construct new efficiency scores after filtering out the main effects of the non-discretionary environmental factors as (Bădin, Daraio, & Simar, 2012; Li & Racine, 2003; Li, Miranti, & Vidyattama, 2017):

$$\varepsilon = \frac{\widehat{\theta_{DEA}}(X,Y|Z)-\mu(z)}{\sigma(z)} \quad \textit{Equation 8}$$

The CFA has not been free from critiques. Consider a probabilistic formulation of production process. The conditional distribution of (X, Y) given Z can be described as a conditional survival function:

$$S_{X,Y|Z}(x,y|z)=S_{X,Y|Z}(x,y|z)S_{Y|Z}(y|z). \quad \textit{Equation 9}$$

The Least-square Cross-validation (LSCV) method to select an adequate bandwidth, *h*, in Equation 6 might not provide an optimal bandwidth to estimate the lower bound of the support of $S_{X,Y|Z}(x,y|z)$ (Bădin *et al*., 2019). This condition introduces a sort of localization bias that is of order ||h|| if the separability condition is violated. Since we cannot know whether the separability condition holds or not *a priori* with real data, the LSCV might not provide an optimal bandwidth (Mastromarco *et al*., 2022). Mastromarco *et al*. (2022) extend the location-scale model provided by Florens *et al*. (2014) to provide cleaned versions of X and Y, or “pure” input and output factors whitened from Z. Then, in this pure input-output space, they define an efficent frontier to estimate a pure measure of managerial efficiency. Their simulations show that the new method based on the pure input-output space outperforms CFA estimators as introduced above: the new method provides estimated efficiency scores closer to true efficiency scores. However, as the sample number approaches from 100 to 500, CFA estimators perform as good as the new method does when the number of Z variables increases (e.g., univariate to bivariate environmental variables). In this paper, the number of DMUs (e.g., local Ohio school districts) is larger than

600, along with four Z variables. Therefore, the final conclusion in this paper should not be affected significantly although applying the new method for future studies is warranted.

3 Two Different Types of Non-discretionary Environmental Factors

Bădin, Daraio, & Simar (2012) and Simar & Wilson (2011) provide succinct examples of two different types of non-discretionary environmental factors, which are slightly modified here:

$$Y = g(X)e^{-U|Z_1-2|} \quad \textit{Equation 10}$$

$$Y = g(X)(1+\frac{|Z_2-2|}{2})^{\frac{1}{2}}e^{-U} \quad \textit{Equation 11}$$

, where Y is an output, is X an input, U is a one-sided inefficiency process, Z_1 and Z_2 are non-discretionary enviromental factors, and $g(X)$ is a certain production function that links input to output. Equation 10 indicates that Z_1 only affects inefficiencies, thus satisfying the separability condition (Assumption 1). In contrast, Equation 11 exemplifies a violation of the separability condition: Z_2 affects only the boundary of the attainable set of production, (X, Y), by shifting the level of the attainable frontier. The separability is confined primarily to an environmental variable like Z_2. There is an environmental factor that affects both inefficiencies and production settings as in Equation 12.

$$Y = g(X)(1+\frac{|Z_3-2|}{2})^{\frac{1}{2}}e^{-U|Z_3-2|} \quad \textit{Equation 12}$$

Studies have shown various examples of Z_3 in local school distrits. For instance, monitoring force tends to *directly* affect the efficiency of administration and teaching in school districts (Duncombe *et al*., 1997; Duncombe and Yinger, 2001, 2011; Eom *et al*., 2014). Higher tax burden induces taxpayers to monitor school district operation tightly such that school districts enhance the efficiency of administration and teaching. If taxpayers are wealthy, they are less likely to monitor school district operation tightly. Parents who have children attending K-12 schools tend to monitor the efficiency of administration and teaching in school districts especially when their tax burden is high. They are likely to monitor the efficiency more tightly than those without school-age children or those with much lower tax burden. As a result, the efficiency is likely to increase.

The above non-discretionary environmental factors for school districts, which influence the “monitoring” force of parents and directly affect efficiency, are different from hostile or favorable production settings that *indirectly* affect efficiency by shifting production settings or frontiers. For instance, Ray’s (1991) method adjusts efficiency scores for the percentage of students from single-parent families as one variable measuring hostile production settings. In the literature on education policy, this variable tends to tap hard-to-teach student groups that of course indicate hostile or unfavorable educational settings for administration and teaching. Under the hostile production settings, however, administration and teaching may or may not be efficient although educational outputs may be generally lower. In other words, educational outputs under the hostile production settings may be lower in general but we cannot make any judgment of the efficiency of administration and teaching yet.

Ray (1991) adjusts for per capita income as a measure of environmental condition. Per capita income is usually regarded as favorable production settings for educational institutions, which may shift production frontiers for the institutions. However, higher levels of income generally lead to lax monitoring over DMUs’ management and as a result, efficiency is supposed to decrease (Eom *et al*., 2014). Thus, income exemplifies a *mixed* environmental factor. Ruggiero and Vitaliano (1999) also mix up the two subtly different sets of environmental factors in their regression adjustment. One of their environmental factors is the percentage of school-age children, for instance. The variable is likely to tap the monitoring force over school administration and teaching, which may come from parental attention to their children. Therefore, percentage of school-age children may heighten efficiency of administration and teaching in school districts. At the same time, however, the percentage might still be deemed as some extra educational burden for educational institutions. If so, this variable, as unfavorable production settings, might also affect production frontiers for the institutions as well as their efficiency.

The separability condition indicates that the environmental factors should affect the production process of DMUs only through the conditional density, $f_{XY|Z}(x, y|z)$, and should not affect its support, Ψ^z. However, the separability test is somewhat unclear about the third venue, through which the environmental factors affect the production process. What if the *mixed* environmental factors affect the process through *both* the conditional density and its support? Empirical findings in this paper clearly show that we should not use the *mixed* environmental factors *alone* in the first-stage CFA analysis as the following section indicates.

4 Findings and Conclusions

Data for 607[1] local Ohio school districts from School Year (SY) 2016 to SY 2019 come from a series of *District Profile Report*[2] and *Ohio School Report Cards*.[3] All variables are in natural log to scale them into similar ranges.[4] There is one factor in the output vector, Y: Performance Index Score (PIS). PIS is a composite proficiency measure of students' academic performance for school districts. There are three factors in the input vector, X: Teachers Average Salary, Administrators Average Salary, and Building Operation Expenditure Per Pupil. There are four factors in the non-discretionary environmental factors vector, *Z:* White Students as Percent of Total Students, Percent of Students with Disability, Per Pupil Revenue Raised by One Mill, and Local Tax Effort Index.

The four environmental factors are slightly different in their nature. The first two factors, White Students as Percent of Total Students and Percent of Students with Disability, exemplify "hostile/favorable production settings," which affect production sets or frontiers. White Students as Percent of Total Students generally tap favorable production settings for students' academic performance while Percent of Students with Disability taps less favorable or hostile production settings in school districts: they exemplify Z_2 in Equation 11.

Per Pupil Revenue Raised by One Mill measures property wealth of school districts, which shows the fiscal capacity of school districts to raise property tax revenue by one mill of taxes, measured per pupil. The previous studies define wealth-related factors as monitoring force from external environments. Therefore, a higher level of Per Pupil Revenue Raised by One Mill might lead to lax monitoring of school district administration and teaching because taxpayers might not be strongly incentivized to monitor school operation when there are abundant resources. As a

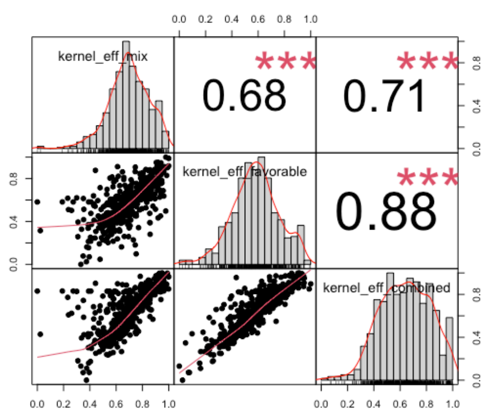
[1] There are 605 observations for SY 2016.
[2] http://education.ohio.gov/Topics/Finance-and-Funding/School-Payment-Reports/District-Profile-Reports [accessed April 25, 2021]
[3] https://reportcard.education.ohio.gov/download [accessed April 25, 2021]
[4] When all the logged variables are scaled such that they range from zero to one, the empirical results are identical to those reported in this paper.

result, abundant fiscal resources tend to lower efficiency of school district operation. However, this wealth factor might also function as favorable production settings as the abundant resources might provide better educational environments.

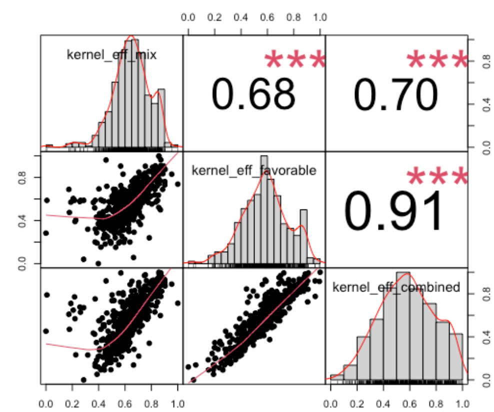
Local Tax Effort Index intends to measure the extent of the effort school district taxpayers make in supporting public K-12 education. Thus, this factor might tap some monitoring force from the taxpayers over school district administration and teaching. However, this index is based on school district income tax and some property taxes, linked to the federal adjusted gross income.[5] As a result, this index might also measure the fiscal capacity of school districts, which reflects hostile/favorable production settings for school districts. Per Pupil Revenue Raised by One Mill and Local Tax Effort Index are good examples of the *mixed* environmental factors that might affect *both* the conditional density, $f_{XY|Z}(x, y|z)$, and its support, Ψ^z in the production process of DMUs: they exemplify Z_3 in Equation 12.

Figure 1: Correlations among Different Efficiency Measures

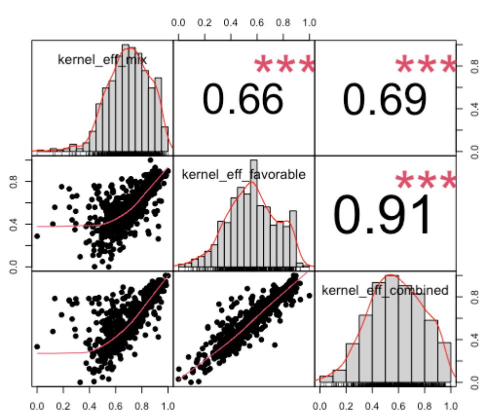
Year 2016



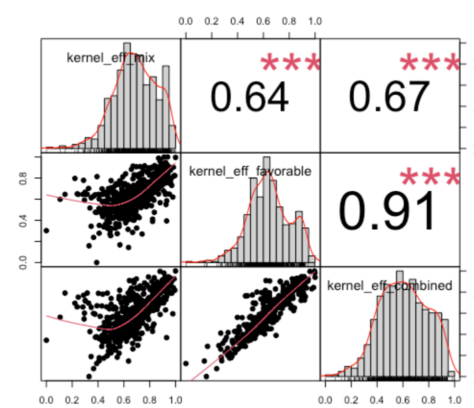
Year 2017



Year 2018



Year 2019



Note: *** ($p < 0.001$), ** ($p < 0.01$), * ($p < 0.05$), . ($p < 0.1$)

[5] The formula is too complex to report here. Readers are referred to the URL for *District Profile Report* for the details of the formula.

Based on the methods in Section 2, conditional DEA efficiency scores (i.e., Equation 8) after whitening out the main effect of environmental factors are obtained for three sets of Z variables: 1) White Students as Percent of Total Students and Percent of Students with Disability (hostile/favorable production settings), 2) Per Pupil Revenue Raised by One Mill and Local Tax Effort Index (mixed environmental factors), and 3) all four variables (combined factors).[6]

Figure 1 shows Pearson correlation coefficients among three efficiency measures: kernel_eff_favorable (#1 above), kernel_eff_mix (#2 above), and kernel_eff_combined (#3 above).[7] Since kernel_eff_combined is efficiency scores that account for all four environmental variables, we can deem the scores as the least biased efficiency measures under this setting. kernel_eff_favorable is strongly correlated with kernel_eff_combined: the correlation coefficients are mostly 0.91 for the four years. If conditional efficiency measures account for favorable/hostile environmental factors, they are very close to the least biased efficiency measures. In contrast, kernel_eff_mix is not as strongly correlated with the latter as kernel_eff_favorable is: the correlation coefficients range between 0.64 and 0.71. The mixed environmental factors also violated the separability condition, and the attendant conditional efficiency scores are supposed to measure true efficiency scores. However, the efficiency scores based on the mixed environmental factors deviates from the least biased efficiency scores. This finding clearly indicates that we should not use the *mixed* environmental factors *alone* in the first stage of the CFA analysis. Instead, we should adjust for hostile/favorable production settings at the minimum when we apply CFA as the separability condition itself implies.

References

Agasisti, T., & Bonomi, F. (2014). Measuring the "Managerial" Efficiency of Public Schools: A Case Study in Italy. *International Journal of Educational Management, 28*(2), 120-140.

Bădin, L., Daraio, C., & Simar, L. (2012). How to Measure the Impact of Environmental Factors in a Nonparametric Production Model. *223*(3), 818-833.

Bădin, L., Daraio, C., & Simar, L. (2019). A Bootstrap Approach for Bandwidth Selection in Estimating Conditional Efficiency Measures. *European Journal of Operational Research, 277*(2), 784-797.

Banker, R. D., & Morey, R. C. (1986). Efficiency Analysis for Exogenously Fixed Inputs and Outputs. *Operations Research, 34*(4), 513-521.

Bifulco, R., & Bretschneider, S. (2001). Estimating School Efficiency: A Comparison of Methods Using Simulated Data. *Economics of Education Review, 20*(5).

Bifulco, R., & Bretschneider, S. (2003). Response to Comment on Estimating School Efficiency. *Economics of Education Review, 22*(6), 635-638.

Chakraborty, K., & Blackburn, V. (2013). Efficiency and Equity in Funding for Government Schools in Australia. *Australian Economic Papers, 52*(3/4), 127-142.

Chakraborty, K., & Poggio, J. (2008). Efficiency and Equity in School Funding: A Case Study for Kansas. *International Advances in Economic Research, 14*(2), 228-241.

Daraio, C., & Simar, L. (2005). Introducing Environmental Variables. *Journal of Productivity Analysis, 24*(1), 93-121.

[6] Kolmogorov-Smirnov (KS) test results show that the separability assumption is violated for all these three sets of efficiency scores for all four years.

[7] These results are virtually identical to those from full frontier estimates because m=600 in order-m frontier analysis. When partial frontier analysis was conducted (i.e., m=300), the final correlation coefficients were almost unchanged.

Daraio, C., & Simar, L. (2007a). *Advanced Robust and Nonparametric Methods in Efficiency Analysis: Methogology and Applications.* New York City, NY: Springer.

Daraio, C., & Simar, L. (2007b). Conditional Nonparametric Frontier Models for Convex and Nonconvex Technologies: A Unifying Approach. *Journal of Productivity Analysis, 28*(1), 13-32.

Daraio, C., Simar, L., & Wilson, P. (2021). Quality as a Latent Heterogeneity Factor in the Efficiency of Universities. *Economic Modeling, 99*, 1-17.

Daraio, C., Simar, L., & Wilson, P. W. (2018). Central Limit Theorems for Conditional Efficiency Measures and Tests of the 'Separability' Condition in Non-parametric, Two-stage Models of Production. *The Econometrics Journal, 21*(2), 170-191.

De Witte, K., & López-Torres, L. (2017). Efficiency in Education: A Review of Literature and A Way Forward. *The Journal of the Operational Society, 68*(4), 339-363.

Debreu, G. (1951). The Coefficient of Resource Utilization. *Econometrica, 19*(3), 273-292.

Dong, Y., Hamilton, R., & Tippett, M. (2014). Cost Efficiency of the Chinese Banking Sector: A Comparison of Stochastic Frontier Analysis and Data Envelopment Analysis. *Economic Modeling, 36*.

Duncombe, W. D., & Yinger, J. (1997). Why Is It So Hard to Help Central City Schools? *Journal of Policy Analysis and Management, 16*(1), 85-113.

Duncombe, W. D., & Yinger, J. (2001). Alternative Paths to Property Tax Relief. In W. Oates (Ed.), *Property Taxation and Local Government Finance* (pp. 243-294). Cambridge, MA: Lincoln Institute of Land Policy.

Duncombe, W. D., & Yinger, J. (2011). Making Do: State Constraints and Local Responses in California's Education Finance System. *International Tax and Public Finance, 18*(3), 337-368.

Duncombe, W. D., Miner, J., & Ruggiero, J. (1997). Empirical Evaluation of Bureaucratic Models of Inefficiency. *Public Choice, 93*(1-2), 1-18.

Eom, T. H., Nguyen-Hoang, P., & Yinger, J. (2014). The Unintended Consequences of Property Tax Relief: New York's STAR Program. *Education Finance and Policy, 9*(4), 446-480.

Farrell, M. J. (1957). The Measurement of Productive Efficiency. *Journal of the Royal Statistical Society. Series A (General), 120*(3), 253-290.

Farsi, M., & Filippini, M. (2005). *A Benchmarking Analysis of Electricity Distribution Utilities in Switzerland.* Zürich, Switzerland : Centre for Energy Policy and Economics, Swiss Federal Institutes of Technology.

Florens, J.-P., Simar, L., & Van Keilegom, I. (2014). Frontier Estimation in Nonparametric Location-scale Models. *Journal of Econometrics, 178*, 456-470.

Huguenin, J.-M. (2015). Adjusting for the Environment in DEA: A Comparison of Alternative Models Based on Empirical Data. *Socio-Economic Planning Sciences, 52*(December), 41-54.

Lee, B. L., Worthington, A., & Wilson, C. (2019). Learning Environment and Primary School Efficiency: A DEA Bootstrap Truncated Regression Analysis. *International Journal of Educational Management, 33*(4), 678-697.

Li, J., Miranti, R., & Vidyattama, Y. (2017). What Matters in Education: A Decomposition of Educational Outcomes with Multiple Measures. *Educational Research and Evaluation: An International Journal on Theory and Practice, 23*(1-2), 3-25.

Li, Q., & Racine, J. (2003). Nonparametric Estimation of Distributions with Categorical and Continuous data. *Journal of Multivariate Analysis, 86*(2), 266-292.

Mastromarco, C., Simar, L., & Van Keilegom, I. (2022). *Estimating Nonparametric Conditional Frontiers and Efficiencies: A New Approach.* Louvain Institute of Data Analysis and Modeling in economics and statistics.

Mortmier, D. (2002). *Competing Methods for Efficiency Measurement: A Systematic Review of Direct DEA vs SFA/DFA Comparisons.* West Heidelberg, Vic, Australia: Centre for Health Program Evaluation.

Nguyen, P. H., & Pham, D. (2020). The Cost Efficiency of Vietnamese Banks - the Difference between DEA and SFA. *Journal of Economics and Development, 22*(2), 209-227.

Nguyen, T., Nghiem, S., Roca, E., & Sharma, P. (2016). Bank reforms and efficiency in Vietnamese banks: evidence based on SFA and DEA. *Applied Economics, 48*(30), 2822-2835.

Odeck, J. (2007). Measuring Technical Efficiency and Productivity Growth: A Comparison of SFA and DEA on Norwegian Grain Production Data. *Applied Economics, 39*(20), 2617–2630.

Pevcin, P. (2014). Efficiency Levels of Sub-national Governments: A Comparison of SFA and DEA Estimations. *The TQM Journal, 26*(3), 1754-2731.

Ray, S. C. (1991). Resource-Use Efficiency in Public Schools: A Study of Connecticut Data. *Management Science, 37*(12), 1620-1628.

Ruggiero, J. (1996). On the Measurement of Technical Efficiency in the Public Sector. *European Journal of Operational Research, 90*(3), 553-565.

Ruggiero, J. (1998). Non-discretionary Inputs in Data Envelopment Analysis. *European Journal of Operational Research, 111*(3), 461-469.

Ruggiero, J. (2003). Comment on Estimating School Efficiency. *Economics of Education Review, 22*(6), 631-634.

Ruggiero, J., & Vitaliano, D. (1999). Assessing the Efficiency of Public Schools Using Data Envelopment Analysis and Frontier Regression. *Contemporary Economic Policy, 17*(3), 321-331.

Simar, L., & Wilson, P. W. (2007). Estimation and Inference in Two-stage, Semi-parametric Models of Production Processes. *Journal of Econometrics, 136*(1), 31-64.

Simar, L., & Wilson, P. W. (2011). Estimation and Inference in Nonparametric Frontier Models: Recent Developments and Perspectives. *Foundations and Trends in Econometrics, 5*(3-4), 183-337.

Simar, L., & Wilson, P. W. (2011). Two-stage DEA: Caveat Emptor. *Journal of Productivity Analysis, 36*, 205-218.

Simar, L., Vanhems, A., & Van Keilegom, I. (2016). Unobserved Heterogeneity and Endogeneity in Nonparametric Frontier Estimation. *Journal of Econometrics, 190*(2), 360-373.

Theodoridis, A., & Anwar, M. (2011). A Comparison of DEA and SFA Methods: A Case Study of Farm Households in Bangladesh. *The Journal of Developing Areas, 45*(1), 95-110.

Welde, M., & Odeck, J. (2011). The Efficiency of Norwegian Road Toll Companies. *Utilities Policy, 19*(3), 162-171.